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A MONTE-CARLO STUDY OF CONJUNCTION ANALYSIS USING PARAMAT

Darrel J. Conway*

This study uses the numerical engine in the General Mission Analysis Tool, driven from the parallel processing tool Paramat, to model a conjuction analysis between two spacecraft on eccentric, nearly coincident trajectories. The spacecraft initial states are separated by 92 meters, and come to within about 9 meters of each other two days later when propagated on their nominal trajectories. The covariance matrix of the initial state data is used to perturb each spacecraft, and the spacecraft are then propagated to the point of closest approach. A Monte Carlo study of the close approach separations and the probability of collision is presented using these perturbed states. The modeling is performed using several different force models, and the results of each configuration are shown to be similar. Two additional test cases are also briefly examined. Performance data for the study is presented, along with a discussion of the methodology and of the tools used.

INTRODUCTION

Thinking Systems is developing a parallel processing system designed to handle analysis problems that require many runs of a spacecraft mission to generate useful analysis results. That tool, Paramat, has been used to study Monte Carlo runs of spacecraft missions in both Earth orbit[1] and in deep space regimes[2] on a user's workstation, and has shown that for certain problems, analysis tasks can be performed quickly and easily that otherwise require large amounts of analysis time and excessive run result data management. Paramat uses the General Mission Analysis Tool (GMAT) to provide the spacecraft simulation capabilities used in its modeling. It is built to run with the core GMAT numerical engine, unchanged from the released code base, and includes the full functionality of GMAT running in a parallel processing environment.

This document describes a small study[3] using Paramat to assess its usefulness for a conjunction risk analysis problem. The work described here entails the analysis of a selected conjunction problem based on a documented Monte Carlo analysis problem described by S. Alfano.[4] The conjunction problem receiving the focus of this document is described as a stressing case involving two satellites traveling along nearly coincident, highly eccentric Earth centered orbits. The spacecraft start separated by 92 meters, and propagate through a series of orbits until they reach a point of closest approach of 8.6 meters two days later. Alfano provides both the initial spacecraft states and the state covariance matrix, making configuration for GMAT scripting and Paramat runs straightforward to implement.

Using these data, Paramat has been configured to perform large scale Monte Carlo analysis of the close approach scenario described by Alfano as Case 10 in the referenced paper. The purpose of this study is to demonstrate the use of the Paramat/GMAT systems for large scale analysis problems

*Senior Scientist, Thinking Systems, Inc., Tucson, Arizona.

in the conjunction analysis domain, to improve on the modeling used by Alfano in the original study, and to present the performance gains possible using a system like Paramat for problems that become difficult to model with a single application program due to the combinatorial nature of the time required to collect a statistically significant data sample for analysis.

PARAMAT

The parallel mission analysis tool, Paramat, is a parallel processing tool designed to drive many instances of spacecraft modeling software simultaneously over an ethernet backbone. The tool defines a messaging system that uses the industry standard Message Passing Interface (MPI) to launch simulations on the modeling software. That software passes data generated during a run through the messaging interface to Paramat. Paramat receives the data and processes it, producing tables and graphs of the generated data as it is produced. Current Paramat builds use the General Mission Analysis Tool, GMAT, as its numerical engine. GMAT's plug-in infrastructure and its modular, open source design makes it the ideal engine for data generation for Paramat. Any mission that can be scripted in the GMAT system can be run on parallel processes in Paramat. Paramat adds features to GMAT to allow for data passing and for run time mission parameter updates through a collection of GMAT plugin commands. These commands enable Monte Carlo runs of the system as described in this paper, and allow for other multiple process runs for other types of large scale analysis.

The build of Paramat used in this study runs the GMAT R2016a release as its core numerical engine. Paramat replaces the GMAT user interface with an MPI based front end that communicates with the Paramat control program over the host computer's Ethernet backbone. The Paramat control program manages multiple instances of the MPI enabled GMAT system, running a single GMAT executable on each launched process. Paramat launches GMAT scripts on each process, manages communications with the processes, and schedules new runs as GMAT reports that the running script has completed execution. Each MPI enabled GMAT process is started when Paramat initializes, and remains running until Paramat is closed. The MPI processes support the full GMAT system, including all core system capabilities and plug-in modules. Native GMAT test scripts running in Paramat produce results identical to those produced in GMAT when built using the same tool suite, run on the same platform. Paramat validation follows the same rigor as is seem in GMAT validation.[5]

Paramat is under development on Linux based workstations. The Paramat system has been built on Windows and Mac workstations in the past, and will continue to be built on those systems as the tool nears marketability. The current focus on Linux builds of the system is a matter of convenience. All of the tools used to build the system run on all three platforms, and earlier builds of Paramat have been demonstrated on all three platforms.

Figure 1 shows a representative (short) Paramat run for one of the conjunction analysis scenarios described in this paper. The system runs a GMAT script, enhanced to perturb the initial state of each spacecraft based on the state covariances, and accumulates data specified by the user in the script. The script can specify data accumulated in a results window and in one or more plot windows. At the end of a run, statistics can be generated for the accumulated data results, and then displayed to the user. Data can also be displayed in histogram format, showing the count of each data range for a selected data group. Finally, the cumulative probability of collision can be plotted as data is collected, as is seen in the figure. All of the collected data can be written to data files for further analysis. Additionally, the log files for each GMAT process are collected in the Paramat GUI and

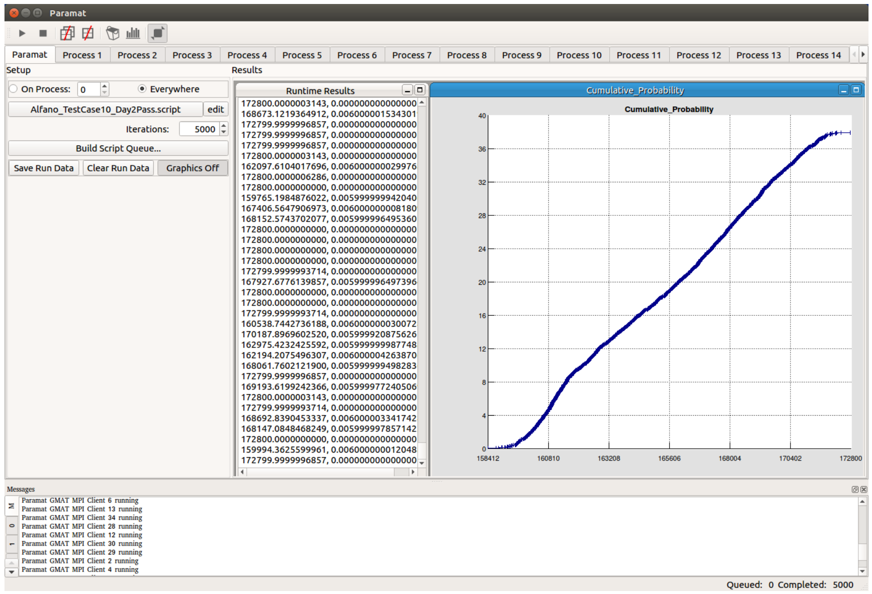

Figure 1. Paramat

can be viewed in the message window docked at the bottom of the Paramat screen during runs or after the system run is completed. GMAT XY plots can also be displayed, on request, in the Paramat GUI in tabbed panels associated with each running process.

METHODOLOGY

The test cases used for the work presented here are described in Alfano.[4] Test case 10 was used for the case study in the original work, performed for the Conjunction Assessment Risk Analysis (CARA) group at NASA Goddard Space Flight Center, and will receive the bulk of the treatment here. Test cases 1 and 4 have been used to further evaluate the performance of the system. The test cases described in Alfano's work include initial state data, covariance matrices, and matching data sets at the conjuction points predicted from the initial state data. His paper presents the results of Monte Carlo analysis of the cumulative probability of impact when propagating from the initial state data, perturbed using the initial covariances and propagated using an analytic universal variable propagator to evaluate the impact probability for each test case, and compares those results to the impact probability obtained using other methods.

In this study, the initial data presented by Alfano is used to seed sets of Monte Carlo runs that use precision numerical integrators to generate the impact probability. The initial state covariances were used to perturb the initial Cartesian state elements for each spacecraft. The spacecraft were then propagated together until either the separation between spacecraft was less than the sum of the radii of the spacecraft, or a maximum propagation time had been reached.

In order to apply perturbations, the rotation matrix that diagonalizes each of the initial state covariance matrices was computed and used to rotate the covariance matrix into diagonal form. The diagonal elements were used to set the standard deviation of a Gaussian noise envelope for each element of a vector representing the perturbations to be applied. The noise applied was a Gaussian, zero mean perturbation set to use the standard deviations culled from the diagonalized covariance matrix. That vector was then rotated back into the Cartesian frame, and applied to the initial Cartesian state for each spacecraft.

The spacecraft propagation was set to use one of GMAT's precision numerical integrators and a force model that can include full field gravitational forces, third body gravitation, atmospheric drag, and solar radiation pressure. Propagation was set to stop either on impact or after a maximum propagation span, as described above. This basic configuration was coded into Paramat scripts, and used for each of the test cases described below.

TEST PROBLEMS

Three test problems are described below. The bulk of the discussion is focussed on Alfano test case 10, showing first that the Paramat system, using precision propagation, reproduces the results seen by Alfano's results generated using an analytic two-body propagator, and then extending examination of the problem outside of the time span that Alfano considered. Test cases 1 and 4 from Alfano's paper are then briefly discussed as further validation of Paramat as a tool for conjunction analyses.

Alfano Test Case 10

Alfano's test case 10 models two spacecraft in highly eccentric (eccentricity = 0.74, semimajor axis = 26553 km), nearly coincident orbits. The nominal trajectories, modeled in GMAT and shown in Figure 2, propagate for two days, resulting in a close approach separation of 8.62 meters at conjunction using the force model configuration employed for this study. The spacecraft traverse four orbits during the two day propagation to the closest approach point. The conjunction point for the two spacecraft in this model is near the fourth apogee point, occurring at a true anomaly of 187.2 degrees. The initial conditions for the spacecraft are given in Table 1, truncated from the full state data found in the appendix of Alfano's paper. (The full precision state and covariance data reported by Alfano were used for the actual modeling documented here.)

Table 1. Initial Earth-centered MJ2000 Cartesian States for the Spacecraft

	Primary	Secondary
Epoch (TAI)	1/1/2000 12:00	1/1/2000 12:00
X (km)	-4725.170	-4725.255
Y (km)	20292.000	20291.985
Z (km)	40356.010	40355.978
Vx (km/s)	-1.4631551	-1.4631551
Vy (km/s)	-0.2635036	-0.2635083
Vz (km/s)	-0.5756667	-0.5756761

The problem studied here examines the effects of initial state imprecision on the two spacecraft in order to estimate the probability that the spacecraft will collide at the point of closest approach. The methodology uses the state data from Table 1 along with the covariance data shown in Table 2, and perturbs the initial state data based on those covariances. The results correspond to test case 10 documented in Alfano.[4] One validation of the methodology presented here is the reproduction of the cumulative probability over time presented by Alfano.

For the purposes of the results presented here, propagation was performed using GMAT's Runge-Kutta 8(9) orbit propagator, with a force model that includes a 4x4 JGM3 Earth gravitational field, and perturbations from the Sun, Moon, and solar radiation pressure. Paramat was used to model

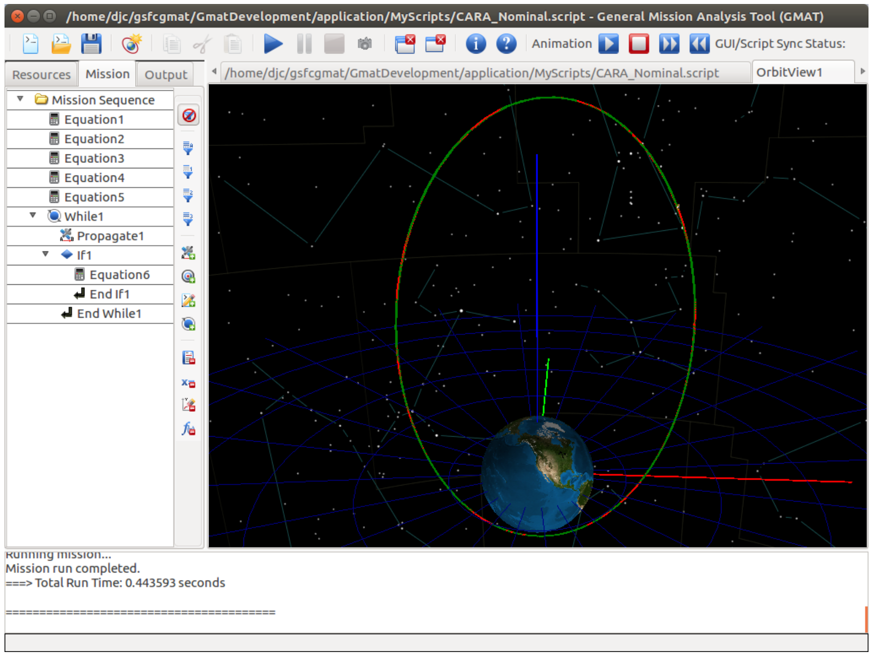

Figure 2. Test Case 10 Orbit

this trajectory through 200,000 iterations in order to build data sets for analyzing the probability of collision between the spacecraft. Propagation for this initial set of runs proceded in two steps: an initial 1.75 day propagation advanced the spacecraft from the initial state to the start of the orbit regime studied, followed by propagation for half a day that includes evaluation of the impact separation criterion. This two phase propagation matched the modeling performed in the reference paper, where the impact probability was evaluated over a half day time span centered on the nominal point of closest approach two days after the epoch of the initial state data.

A second Paramat run of 200,000 iterations was performed using an Earth point mass only force model, in order to examine differences in the impact statistics rising from differences in the simulation dynamics.

Table 2. State Covariances at Initial Epoch

Primary Spacecraft					
0.082114838	0.007584577	0.016569746	0.0	0.0	0.0
0.007584577	0.041365927	0.002984091	0.0	0.0	0.0
0.016569746	0.002984091	0.046519234	0.0	0.0	0.0
0.0	0.0	0.0	1 x 10^{-8}	0.0	0.0
0.0	0.0	0.0	0.0	1 x 10^{-8}	0.0
0.0	0.0	0.0	0.0	0.0	1 x 10^{-8}
Secondary Spacecraft					
0.082114617	0.007584674	0.016569931	0.0	0.0	0.0
0.007584674	0.041365969	0.002984178	0.0	0.0	0.0
0.016569931	0.002984178	0.046519414	0.0	0.0	0.0
0.0	0.0	0.0	1 x 10^{-8}	0.0	0.0
0.0	0.0	0.0	0.0	1 x 10^{-8}	0.0
0.0	0.0	0.0	0.0	0.0	1 x 10^{-8}

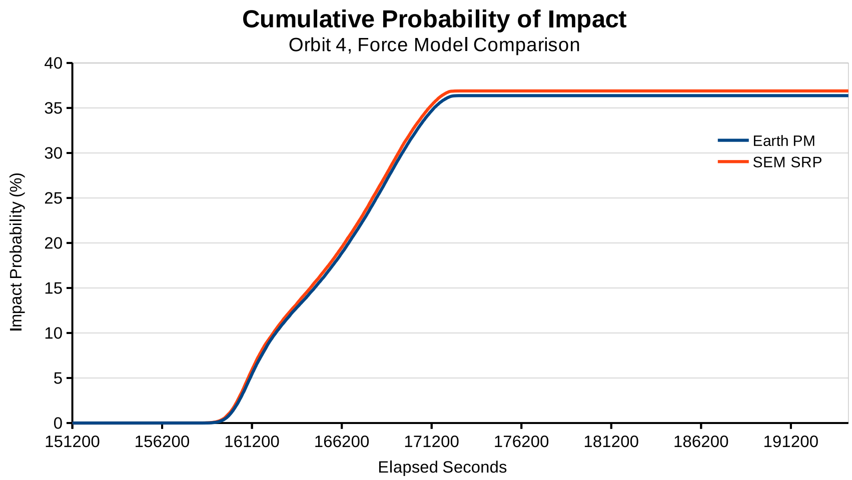

Figure 3. Impact Probability, Full Force vs Point Mass

Figure 3 shows the cumulative impact probability over time for these two runs. The probability curves shown here qualitatively match the probability curve shown in Alfano. Table 3 shows the net cumulative probability in the Paramat runs along with Alfano's results. The probability of impact estimated by Paramat can be seen to be in line with the estimates made by Alfano's Monte Carlo runs.

Table 3. Test Case 10 Cumulative Probability of Impact

Source	Iterations	Probability (%)	Run Time (s)
Alfano	1,000,000	36.6415000	—
Alfano	100,000,000	36.2952470	—
Paramat, Point Mass Only	200,000	36.3621818	2465.30
Paramat, Earth 4x4, Sun, Moon, SRP	200,000	36.6294157	3365.92

Alfano Test Case 10, Extended

One wrinkle found early in the evaluation of test case 10 was the likelihood that the spacecraft would impact before ever reaching the fourth orbit of the spacecraft trajectories. Figure 4 shows the spacecraft to spacecraft range for 23 perturbed orbits for test case 10, generated in Paramat. The half day span surrounding the nominal point of closest approach is marked on the figure with a box labeled "Conjunction Region." The figure shows that there is a non-zero likelihood that the spacecraft will collide outside of that time span, with many potential impacts between the spacecraft occurring before the spacecraft ever enter the fourth orbit examined in the original problem analysis. The presence of these earlier potential impacts necessitated the two step propagation scheme described above in order to reproduce the results in the literature. GMAT scripting for this problem is simplest when evaluating the interspacecraft separation throughout the orbit propagation. That configuration shows that the effects of the impact likelihood are markedly higher when examining the entire propagation span.

In order to evaluate the statistics for the full trajectory that may contain impacts, the initial 1.75 day orbit propagation was removed from the Paramat script used for test case 10, and the time span

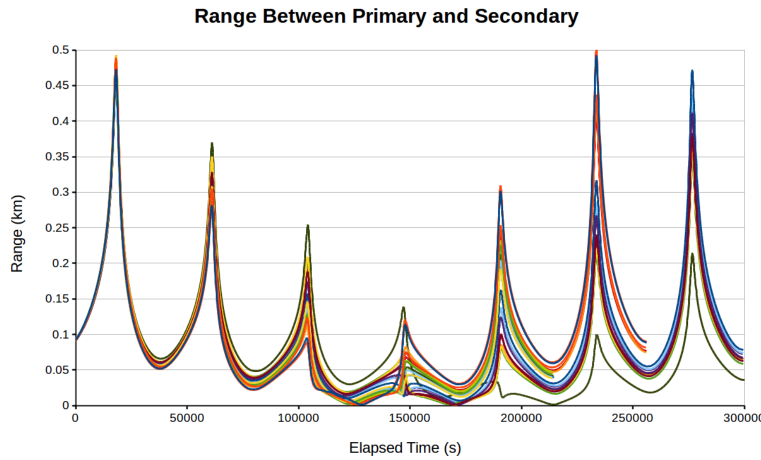

Figure 4. Spacecraft to Spacecraft Range, 23 Cases

used for the evaluation was extended from 2.25 days to 7 days. This span models propagations well past all observed closest approaches seen over the course of this study for test case 10. The orbit preceding the nominal closest approach time was found to be nearly as likely to produce an impact using this configuration, as can be seen in the impact probability plot shown in Figure 5. When runs seeking all impacts over the 7 day period were evaluated, the probability of impact in a 200,000 ieration run before reaching orbit 4 was found to be 36.130%. The impact probability over the full 7 day run was 77.036%, significantly higher than the 36% probability estimated when only examining the region of the fourth orbit. This shows that Paramat can be used to extend analyses like this over larger analysis spans with minimal effort. Additionally, the run time seen for Paramat for the extended propagation period was just under 65 minutes of processing on a 12 core workstation (actual run time: 3881.99 sec) using the extended 7-day region of study. That run time compares favorably with the 56 minute run time needed to analyze the original half day conjunction region.

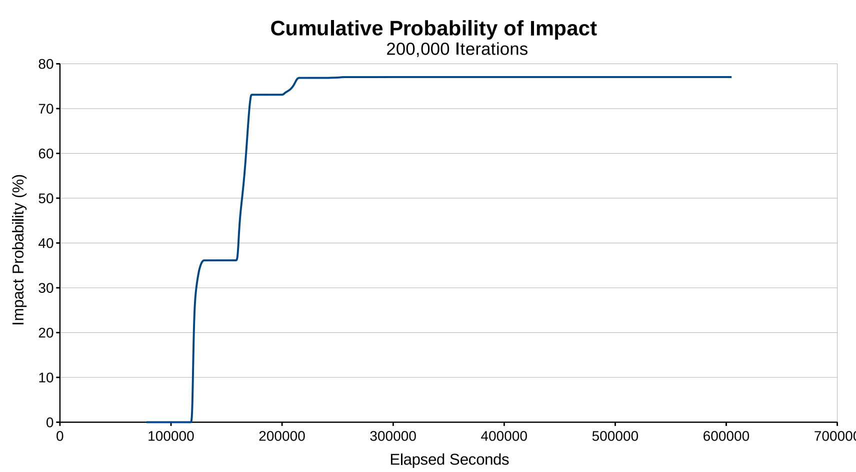

Figure 5. Test Case 10 Impact Probability, 7 Days of Propagation

Test Cases 1 and 4

As final validation of Paramat as a tool for conjunction analysis, test cases 1 and 4 from Alfano's paper were evaluated using Paramat. Test case 1 involves two spacecraft in geosynchronous orbit, with a separation at closest approach inside of the combined spacecraft radii. Test case 4 also uses geosynchronous spacecraft, this time with the closest approach separation outside of the combined satellite radii. Figure 6 shows the cumulative impact probability from Paramat for both of these cases. The probability curves shown here are qualitatively good matches for the results shown in Alfano.

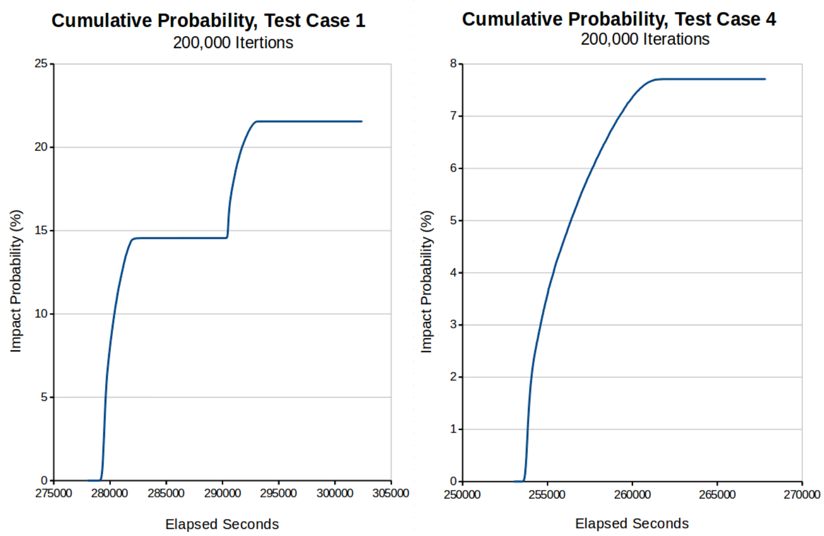

Figure 6. Impact Probability, Test cases 1 (left) and 4

The calculated probabilities from these 200,000 iteration Monte Carlo runs are shown in Table 4, along with the results calculated by Alfano. Paramat's computed likelihood of impact can be seen to be in line with those calculated by Alfano for both of these test cases. The force model used in these runs is the 4x4 Earth gravity model, supplemented by point mass perturbations from the Sun and Moon, and solar radiation pressure using a cannonball model, as was used in the earlier test case 10 analyses.

Table 4. Test Case 1 and 4 Cumulative Probabilities of Impact

Source	Iterations	Probability (%)	Run Time (s)
Test Case 1			
Alfano	1,000,000	21.6602000	—
Alfano	100,000,000	21.7467140	—
Paramat	200,000	21.5520474	3131.72
Test Case 4			
Alfano	1,000,000	7.8974000	—
Alfano	100,000,000	7.3089530	—
Paramat	200,000	7.7101542	2980.31

PERFORMANCE DATA

Tables 3 and 4 of probabilities show performance times for 200,000 iteration runs of Paramat for their test cases, run on a 12 core Intel Xeon based workstation. Once the test problem is defined, a 200,000 iteration Monte Carlo run on this machine typically completes in less than an hour for this type of problem, even when using a realistic force model in the propagation. The longest run for all of these cases completed in about 65 minutes, evaluating test cast 10 over a 7 day propagation using a relatively complete force model. In this section, performance data on a variety of configurations and platforms is presented using a scaled back set of runs to illustrate the speed and results of the modeling that Paramat produces for different hardware configurations.

The data presented in Table 5 uses 5,000 iterations of the extended Alfano test case 10 configuration to measure the performance of comparable runs of Paramat. The force model is a 4x4 JGM3 Earth gravity model, with third body effects from the Sun and Moon and solar radiation pressure using a cannonball area model. The propagation span evaluates impacts throughout the 7 day span of each iteration. While the statistics generated from these runs are not intended to be definitive measures of the impact probability, their values are reported to illustrate the variations seen in impact probabilities for data that is generated from smaller scale Monte Carlo runs of Paramat.

Table 5. Performance Data, Test Case 10, Linux Workstation

Configuration		Laptop		Workstation	
Process Count	Iterations	Time (s)	Prob. (%)	Time (s)	Prob. (%)
1	5000	890.849	76.755	932.040	76.740
2	5000	461.368	76.515	454.471	78.240
4	5000	270.157	77.940	237.255	76.931
6	5000	259.706	77.656	159.008	76.180
8	5000	236.885	76.400	117.237	77.796
12	5000	235.535	76.760	81.625	77.320
16	5000	227.678	77.200	77.384	77.720
20	5000	229.042	77.140	73.850	77.820
24	5000	226.848	77.415	72.087	76.580
36	5000	226.872	76.960	71.375	76.360
48	5000	220.907	77.600	70.393	77.211
60	5000	—	—	70.494	76.520
72	5000	—	—	72.667	76.395
84	5000	—	—	72.674	77.787
100	5000	—	—	83.671	76.980

These data were generated on two platforms: a high end engineering workstation and a smaller laptop computer. The engineering workstation is a dual processor, 12 core Xeon workstation running Ubuntu Linux 16.04. The laptop computer is a quad-core i7 based Linux laptop, also running Ubuntu Linux 16.04. On both machines, Paramat was configured to use the number of GMAT processes indicated in the table, starting from a single process and proceeding until performance began to degrade from oversaturation of the hardware or until the number of processes running was many times the number of cores available for the run. Paramat was restarted for each run.

Figure 7 shows the run time for each Paramat configuration as a function of process count. As can be seen in the figure, performance levels out when the number of GMAT processes matches the number of processing cores available for the run. Increasing the number of processes beyond the processor count does not improve performance for the system significantly, but it also does not degrade performance until the process count becomes many times larger than the core count.

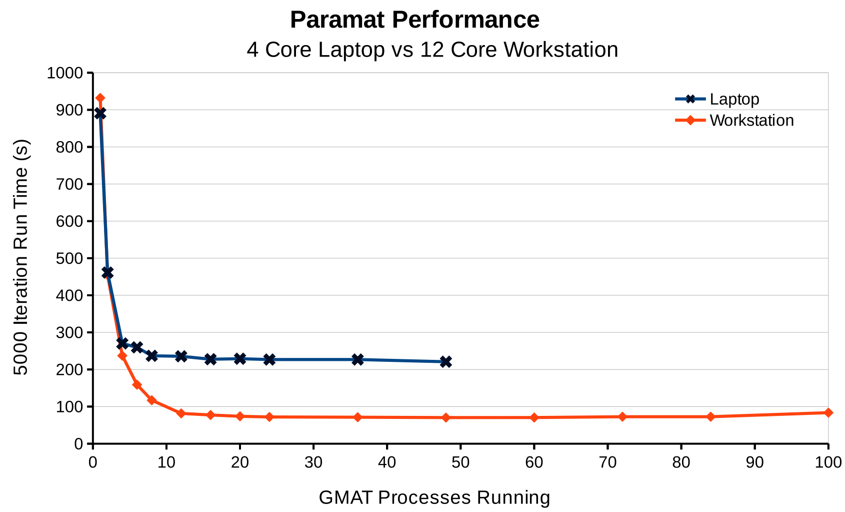

Figure 7. Paramat Run-time Performance

In other words, large scale Monte Carlo runs of conjunction studies using Paramat benefit from additional processing hardware in the workstation used for the analysis, as one would expect for this type of parallelization.

CONCLUSION

Thinking Systems is developing a parallel processing system for use in large scale, many run analysis problems. This system, Paramat, has been demonstrated for use in Monte Carlo studies of conjunction problems, and shown to produce results matching those seen elsewhere in the open literature. Paramat provides a new tool for research groups investigating conjunction scenarios using the proven, operational modeling capabilities of GMAT.

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